

Bin Packing Problem instance results

This document contains an instance wise comparison of the results for two approaches on a set of publicly available instances for the Bin Packing Problem¹. The two approaches are the Benders decomposition based approach of [1] and the Branch-and-Price based procedure of [2].

References

- [1] R. Lusby, M. Gamst, and S. Spoorendonk. *Simultaneously Exploiting Two Formulations: An Exact Benders Decomposition Approach*. DTU Management Engineering, 2016.
- [2] J. Valério de Carvalho. Exact solution of bin-packing problems using column generation and branch-and-bound. *Annals of Operations Research*, 86(0):629–659, 1999.

¹<http://www.math.u-bordeaux1.fr/~fvanderb/publications.html>

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp1a	58	128	4129	48	48	26	13	0	11	0.299	4.013
bp1b	59	127	4132	48	48	1	0	0	0	0.333	0.461
bp1c	61	131	4658	46	46	1	0	0	0	0.407	0.541
bp1d	68	129	4806	49	49	28	14	0	12	0.238	2.168
bp1e	62	127	4226	50	50	1	0	0	0	0.377	0.520
bp1f	61	127	4214	48	48	1	0	0	0	0.346	0.478
bp1g	65	129	4663	48	48	14	7	0	5	0.399	1.377
bp1h	64	123	4023	49	49	16	8	0	6	0.280	1.291
bp1i	67	129	4772	50	50	40	20	0	18	0.421	4.325
bp1j	64	129	4656	46	46	16	8	0	6	0.423	1.759
bp1k	64	126	4231	52	52	1	0	0	0	0.334	0.471
bp1l	60	126	3995	49	49	1	0	0	0	0.256	0.363
bp1m	63	128	4550	48	48	26	13	0	11	0.295	2.273
bp1n	62	127	4284	49	49	1	0	0	0	0.315	0.446
bp1o	61	124	3965	50	50	1	0	0	0	0.397	0.543
bp1p	63	128	4388	48	48	1	0	0	0	0.194	0.284
bp1q	64	123	3976	52	52	3	1	0	0	0.168	0.253
bp1r	64	123	3973	52	52	3	1	0	0	0.120	0.191
bp1s	66	126	4443	49	49	1	0	0	0	0.229	0.319
bp1t	68	130	4953	49	49	60	30	0	28	0.238	5.786
Column avg.				49.00	49.00	12.10	5.75	0.00	4.85	0.30	1.39
Column avg.^a				49.10	49.10	23.20	11.50	0.00	9.70	0.29	2.34

Table 1: binpack1 instances - Benders Decomposition

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{lp}^*	Nodes	Level	Price (s)	Solve (s)
bp1a	58	128	4129	48	48	7	3	0.355	0.692
bp1b	59	127	4132	48	48	1	0	0.339	0.442
bp1c	61	131	4658	46	46	1	0	4.123	4.214
bp1d	68	129	4806	49	49	7	3	0.451	0.895
bp1e	62	127	4226	50	50	1	0	0.239	0.318
bp1f	61	127	4214	48	48	1	0	0.187	0.245
bp1g	65	129	4663	48	48	9	4	0.303	0.695
bp1h	64	123	4023	49	49	17	8	0.358	0.887
bp1i	67	129	4772	50	50	19	9	0.326	1.471
bp1j	64	129	4656	46	46	89	44	0.853	8.215
bp1k	64	126	4231	52	52	1	0	0.244	0.322
bp1l	60	126	3995	49	49	1	0	0.138	0.186
bp1m	63	128	4550	48	48	167	83	2.567	22.872
bp1n	62	127	4284	49	49	1	0	0.174	0.246
bp1o	61	124	3965	50	50	1	0	0.220	0.277
bp1p	63	128	4388	48	48	1	0	0.200	0.258
bp1q	64	123	3976	52	52	3	1	0.173	0.241
bp1r	64	123	3973	52	52	3	1	0.125	0.182
bp1s	66	126	4443	49	49	1	0	0.233	0.310
bp1t	68	130	4953	49	49	71	35	0.734	7.110
Column avg.				49.00	49.00	20.10	9.55	0.62	2.50
Column avg.	^a			49.10	49.10	39.20	19.10	0.62	4.33

Table 2: binpack1 instances - Branch-and-Price [2]

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp2a	71	129	4942	99	99	1	0	0	0	0.565	0.681
bp2b	78	132	5786	100	100	1	0	0	0	0.747	0.897
bp2c	76	130	5455	102	102	1	0	0	0	0.571	0.674
bp2d	79	131	5730	100	100	1	0	0	0	0.592	0.705
bp2e	76	131	5498	101	101	94	47	0	45	0.576	9.878
bp2f	78	131	5639	101	101	22	11	0	9	1.151	3.439
bp2g	78	132	5802	102	102	1	0	0	0	1.021	1.216
bp2h	77	130	5478	103	103	6	3	0	1	0.574	1.093
bp2i	78	130	5549	105	105	16	8	0	6	0.681	2.520
bp2j	80	131	5769	101	101	1	0	0	0	1.038	1.235
bp2k	77	130	5489	105	105	28	14	0	12	0.736	3.436
bp2l	80	131	5777	101	101	150	75	0	73	1.204	18.827
bp2m	76	128	5197	105	105	24	12	0	10	0.518	3.077
bp2n	75	131	5480	103	103	1	0	0	0	0.871	1.052
bp2o	79	132	5821	100	100	1	0	0	0	0.750	0.913
bp2p	78	130	5534	105	105	22	11	0	9	0.648	3.213
bp2q	76	132	5674	97	97	8	4	0	2	1.063	1.829
bp2r	76	131	5565	100	100	1	0	0	0	0.809	0.949
bp2s	76	132	5627	100	100	38	19	0	17	0.756	4.844
bp2t	81	132	5962	102	102	1	0	0	0	1.050	1.257
Column avg.				101.60	101.60	20.90	10.20	0.00	9.20	0.80	3.09
Column avg.^a				102.30	102.30	40.80	20.40	0.00	18.40	0.79	5.22

Table 3: binpack2 instances - Benders Decomposition

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Price (s)	Solve (s)
bp2a	71	129	4942	99	99	1	0	0.501	0.581
bp2b	78	132	5786	100	100	1	0	0.768	0.869
bp2c	76	130	5455	102	102	1	0	0.601	0.682
bp2d	79	131	5730	100	100	1	0	0.617	0.703
bp2e	76	131	5498	101	101	5	2	0.591	0.788
bp2f	78	131	5639	101	101	61	30	1.134	6.716
bp2g	78	132	5802	102	102	1	0	0.548	0.639
bp2h	77	130	5478	103	103	149	74	2.758	19.245
bp2i	78	130	5549	105	105	223	111	2.935	29.863
bp2j	80	131	5769	101	101	1	0	0.761	0.881
bp2k	77	130	5489	105	105	7	3	0.711	1.049
bp2l	80	131	5777	101	101	9	4	0.744	1.178
bp2m	76	128	5197	105	105	137	68	1.736	12.769
bp2n	75	131	5480	103	103	1	0	0.478	0.567
bp2o	79	132	5821	100	100	1	0	0.824	0.943
bp2p	78	130	5534	105	105	47	23	1.012	4.665
bp2q	76	132	5674	97	97	5	2	0.610	0.829
bp2r	76	131	5565	100	100	1	0	0.586	0.672
bp2s	76	132	5627	100	100	181	90	2.417	26.592
bp2t	81	132	5962	102	102	1	0	0.615	0.718
Column avg.				101.60	101.60	41.70	20.35	1.05	5.55
Column avg.^a				102.30	102.30	82.40	40.70	1.46	10.37

Table 4: binpack2 instances - Branch-and-Price [2]

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp3a	81	132	5962	198	198	1	0	0	0	1.184	1.301
bp3b	81	132	5962	201	201	6	3	0	1	1.106	1.724
bp3c	80	132	5883	202	202	3	1	0	0	1.478	1.648
bp3d	81	132	5962	204	204	430	215	0	213	1.496	59.155
bp3e	80	131	5769	206	206	1	0	0	0	1.665	1.848
bp3f	81	132	5962	206	206	1	0	0	0	1.225	1.346
bp3g	81	132	5962	207	207	62	31	0	29	1.017	9.070
bp3h	80	132	5896	204	204	18	9	0	7	1.566	4.024
bp3i	81	132	5962	196	196	42	21	0	19	1.546	6.818
bp3j	81	132	5962	202	202	1	0	0	0	1.515	1.676
bp3k	81	132	5962	200	200	1	0	0	0	1.213	1.339
bp3l	81	132	5962	200	200	70	35	0	33	1.525	9.887
bp3m	81	132	5962	199	199	6	3	0	1	1.452	2.033
bp3n	81	132	5962	196	196	8	4	0	2	1.606	2.492
bp3o	81	132	5962	204	204	1	0	0	0	1.840	2.028
bp3p	80	132	5924	201	201	1	0	0	0	1.169	1.279
bp3q	81	132	5962	202	202	1	0	0	0	1.154	1.274
bp3r	81	132	5962	198	198	1	0	0	0	1.138	1.254
bp3s	81	132	5962	202	202	120	60	0	58	1.111	15.092
bp3t	81	132	5962	196	196	8	4	0	2	1.242	1.998
Column avg.				201.20	201.20	39.10	19.30	0.00	18.25	1.36	6.36
Column avg.^a				200.64	200.64	70.27	35.09	0.00	33.18	1.38	10.36

Table 5: binpack3 instances - Benders Decomposition

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{lp}^*	Nodes	Level	Price (s)	Solve (s)
bp3a	81	132	5962	198	198	1	0	1.395	1.495
bp3b	81	132	5962	201	201	121	60	2.356	15.965
bp3c	80	132	5883	202	202	3	1	1.376	1.515
bp3d	81	132	5962	204	204	371	185	11.548	94.250
bp3e	80	131	5769	206	206	1	0	1.300	1.414
bp3f	81	132	5962	206	206	1	0	1.428	1.531
bp3g	81	132	5962	207	207	155	77	3.057	19.712
bp3h	80	132	5896	204	204	653	326	25.895	305.712
bp3i	81	132	5962	196	196	677	338	18.151	258.562
bp3j	81	132	5962	202	202	1	0	1.350	1.453
bp3k	81	132	5962	200	200	1	0	1.154	1.243
bp3l	81	132	5962	200	200	9	4	1.187	1.598
bp3m	81	132	5962	199	199	199	99	3.398	31.061
bp3n	81	132	5962	196	196	7	3	1.406	1.767
bp3o	81	132	5962	204	204	1	0	1.269	1.360
bp3p	80	132	5924	201	201	1	0	1.141	1.226
bp3q	81	132	5962	202	202	1	0	1.144	1.240
bp3r	81	132	5962	198	198	1	0	1.372	1.477
bp3s	81	132	5962	202	202	7	3	1.131	1.458
bp3t	81	132	5962	196	196	469	234	12.950	141.135
Column avg.				201.20	201.20	134.00	66.50	4.70	44.26
Column avg. ^a				200.64	200.64	242.82	120.91	7.50	79.34

Table 6: binpack3 instances - Branch-and-Price [2]

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp4a	81	132	5962	399	399	1	0	0	0	3.061	3.248
bp4b	81	132	5962	406	406	1	0	0	0	3.217	3.402
bp4c	81	132	5962	411	411	30	15	0	13	2.661	6.186
bp4d	81	132	5962	411	411	48	24	0	22	2.698	9.427
bp4e	81	132	5962	397	397	42	21	0	19	2.892	8.628
bp4f	81	132	5962	399	399	26	13	0	11	2.686	6.028
bp4g	81	132	5962	395	395	3	1	0	0	2.717	2.917
bp4h	81	132	5962	404	404	1	0	0	0	2.401	2.537
bp4i	81	132	5962	399	399	380	190	0	188	2.172	52.241
bp4j	81	132	5962	397	397	8	4	0	2	2.671	3.407
bp4k	81	132	5962	400	400	1	0	0	0	3.193	3.380
bp4l	81	132	5962	401	401	120	60	0	58	3.404	18.735
bp4m	81	132	5962	393	393	1	0	0	0	2.502	2.636
bp4n	81	132	5962	396	396	1	0	0	0	2.300	2.430
bp4o	81	132	5962	394	394	438	219	0	217	2.291	62.701
bp4p	81	132	5962	402	402	48	24	0	22	2.464	9.356
bp4q	81	132	5962	404	404	1	0	0	0	2.535	2.698
bp4r	81	132	5962	404	404	146	73	0	71	2.198	23.014
bp4s	81	132	5962	399	399	1	0	0	0	2.361	2.496
bp4t	81	132	5962	400	400	3	1	0	0	2.289	2.445
Column avg.				400.55	400.55	65.00	32.25	0.00	31.15	2.64	11.40
Column avg.^a				400.83	400.83	107.67	53.75	0.00	51.92	2.60	17.09

Table 7: binpack4 instances - Benders Decomposition

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Price (s)	Solve (s)
bp4a	81	132	5962	399	399	1	0	2.766	2.895
bp4b	81	132	5962	406	406	1	0	3.137	3.283
bp4c	81	132	5962	411	411	15	7	2.770	3.534
bp4d	81	132	5962	411	411	1165	582	72.079	775.549
bp4e	81	132	5962	397	397	601	300	20.168	232.524
bp4f	81	132	5962	399	399	191	95	4.902	29.494
bp4g	81	132	5962	395	395	3	1	2.358	2.493
bp4h	81	132	5962	404	404	1	0	2.424	2.538
bp4i	81	132	5962	399	399	551	275	10.680	147.527
bp4j	81	132	5962	397	397	813	406	41.440	557.709
bp4k	81	132	5962	400	400	1	0	2.129	2.227
bp4l	81	132	5962	401	401	7	3	2.555	2.893
bp4m	81	132	5962	393	393	1	0	2.473	2.588
bp4n	81	132	5962	396	396	1	0	2.538	2.653
bp4o	81	132	5962	394	394	429	214	12.669	102.384
bp4p	81	132	5962	402	402	87	43	3.340	10.828
bp4q	81	132	5962	404	404	1	0	2.058	2.163
bp4r	81	132	5962	404	404	1051	525	63.909	878.095
bp4s	81	132	5962	399	399	1	0	2.127	2.237
bp4t	81	132	5962	400	400	3	1	2.566	2.705
Column avg.				400.55	400.55	246.20	122.60	12.95	138.32
Column avg.^a				400.83	400.83	409.67	204.33	19.95	228.81

Table 8: binpack4 instances - Branch-and-Price [2]

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp5a	50	548	10955	20	20	1	0	0	0	0.325	0.429
bp5b	56	555	12831	20	20	84	39	2	41	0.379	34.139
bp5c	48	548	10562	20	20	1	0	0	0	0.276	0.42
bp5d	49	540	10053	20	20	12	6	0	5	0.285	3.926
bp5e	48	540	9833	20	20	4	2	0	1	0.421	1.534
bp5f	50	547	10788	20	20	1	0	0	0	0.439	0.617
bp5g	51	547	11013	20	20	22	11	0	10	0.547	7.649
bp5h	50	548	10980	20	20	1	0	0	0	0.418	0.595
bp5i	49	545	10626	20	20	1	0	0	0	0.495	0.675
bp5j	49	547	10574	20	20	1	0	0	0	0.330	0.483
bp5k	53	553	11733	20	20	16	8	0	7	0.314	5.851
bp5l	46	540	9571	20	20	1	0	0	0	0.392	0.547
bp5m	53	555	11857	20	20	70	35	0	34	0.441	23.692
bp5n	51	552	11183	20	20	1	0	0	0	0.301	0.454
bp5o	50	552	11153	20	20	1	0	0	0	0.364	0.472
bp5p	51	551	11347	20	20	22	11	0	10	0.375	7.928
bp5q	46	539	9531	20	20	1	0	0	0	0.343	0.489
bp5r	48	536	9930	20	20	1	0	0	0	0.401	0.560
bp5s	50	550	10778	20	20	1	0	0	0	0.320	0.482
bp5t	51	553	11550	20	20	1	0	0	0	0.272	0.371
Column avg.				20.00	20.00	12.15	5.60	0.10	5.40	0.37	4.57
Column avg.	^a			20.00	20.00	32.86	16.00	0.29	15.43	0.39	12.10

Table 9: binpack5 instances - Benders Decomposition

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{lp}^*	Nodes	Level	Price (s)	Solve (s)
bp5a	50	548	10955	20	20	1	0	0.333	0.434
bp5b	56	555	12831	20	20	35	17	3.494	5.729
bp5c	48	548	10562	20	20	1	0	0.205	0.285
bp5d	49	540	10053	20	20	11	5	1.584	1.961
bp5e	48	540	9833	20	20	9	4	0.862	1.076
bp5f	50	547	10788	20	20	1	0	0.245	0.324
bp5g	51	547	11013	20	20	13	6	1.122	1.646
bp5h	50	548	10980	20	20	1	0	0.231	0.311
bp5i	49	545	10626	20	20	1	0	0.288	0.369
bp5j	49	547	10574	20	20	1	0	0.323	0.415
bp5k	53	553	11733	20	20	5	2	0.956	1.199
bp5l	46	540	9571	20	20	1	0	0.214	0.284
bp5m	53	555	11857	20	20	17	8	1.703	2.527
bp5n	51	552	11183	20	20	1	0	0.262	0.348
bp5o	50	552	11153	20	20	1	0	0.296	0.379
bp5p	51	551	11347	20	20	95	32	5.379	10.095
bp5q	46	539	9531	20	20	1	0	0.183	0.259
bp5r	48	536	9930	20	20	1	0	0.220	0.291
bp5s	50	550	10778	20	20	1	0	0.254	0.333
bp5t	51	553	11550	20	20	1	0	0.277	0.359
Column avg.				20.00	20.00	9.90	3.70	0.92	1.43
Column avg.	^a			20.00	20.00	26.43	10.57	2.16	3.46

Table 10: binpack5 instances - Branch-and-Price [2]

^aAll instances that require branching

Instance	Products	Nodes	Edges	Z_{ip}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp6a	86	588	21408	40	40	210	105	1	104	1.508	110.389
bp6b	85	587	20982	40	40	300	150	1	149	1.562	149.887
bp6c	91	593	23081	40	40	294	147	1	146	1.680	162.961
bp6d	86	588	21496	40	40	168	75	1	83	1.705	97.325
bp6e	92	594	23208	40	40	574	98	4	286	1.620	360.524
bp6f	88	590	22046	40	40	290	145	1	144	1.823	154.906
bp6g	86	588	21418	40	40	218	109	1	108	1.731	117.265
bp6h	87	589	21489	40	40	432	142	1	215	1.670	228.767
bp6i	86	588	21177	40	40	240	119	1	119	1.647	124.187
bp6j	85	587	21129	40	40	272	136	3	135	1.640	146.379
bp6k	83	585	20450	40	40	92	46	0	45	1.744	48.260
bp6l	86	588	21591	40	40	646	96	7	320	1.621	361.260
bp6m	82	584	19612	40	40	330	106	1	164	1.764	158.450
bp6n	87	589	21763	40	40	148	74	1	73	1.579	83.143
bp6o	83	585	20656	40	40	180	90	0	89	1.662	90.788
bp6p	81	583	19923	40	40	242	121	0	120	1.369	112.382
bp6q	87	589	21469	40	40	298	149	0	148	1.344	144.743
bp6r	90	592	22323	40	40	144	72	0	71	1.612	89.210
bp6s	85	587	20724	40	40	194	97	0	96	1.731	95.924
bp6t	87	589	21234	40	40	230	115	0	114	1.365	119.280
Column avg.				40.00	40.00	275.10	109.60	1.20	136.45	1.62	147.80

Table 11: binpack6 instances - Benders Decomposition

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Price (s)	Solve (s)
bp6a	86	588	21408	40	40	121	60	25.857	63.409
bp6b	85	587	20982	40	40	93	46	19.972	47.992
bp6c	91	593	23081	40	40	283	78	58.204	186.730
bp6d	86	588	21496	40	40	121	60	20.899	54.963
bp6e	92	594	23208	40	40	187	93	60.684	155.629
bp6f	88	590	22046	40	40	299	119	76.234	226.664
bp6g	86	588	21418	40	40	113	56	28.546	68.030
bp6h	87	589	21489	40	40	125	62	20.526	52.002
bp6i	86	588	21177	40	40	89	44	20.273	43.478
bp6j	85	587	21129	40	40	129	64	23.598	62.390
bp6k	83	585	20450	40	40	55	27	10.401	21.702
bp6l	86	588	21591	40	40	65	32	18.631	44.754
bp6m	82	584	19612	40	40	69	34	10.659	22.213
bp6n	87	589	21763	40	40	117	58	30.126	73.357
bp6o	83	585	20656	40	40	115	57	20.514	46.194
bp6p	81	583	19923	40	40	121	60	18.531	44.209
bp6q	87	589	21469	40	40	239	119	58.225	153.999
bp6r	90	592	22323	40	40	111	55	33.074	85.204
bp6s	85	587	20724	40	40	101	50	22.250	49.156
bp6t	87	589	21234	40	40	123	61	31.391	73.560
Column avg.				40.00	40.00	133.80	61.75	30.43	78.78

Table 12: binpack6 instances - Branch-and-Price [2]

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp7a	134	636	37895	83	83	658	329	2	328	6.187	566.267
bp7b	140	642	40648	83	83	1978	582	23	988	6.836	1861.600
bp7c	139	641	40490	83	83	1158	579	2	578	6.902	1067.070
bp7d	142	644	41058	83	83	1336	476	2	667	6.740	1340.230
bp7e	134	636	37757	83	83	812	396	3	405	6.305	758.416
bp7f	145	647	43028	83	83	1286	537	5	642	8.483	1447.400
bp7g	138	640	39188	83	83	826	413	2	412	7.104	742.715
bp7h	137	639	39085	83	83	986	493	0	492	7.078	848.650
bp7i	139	641	40328	83	83	1020	510	3	509	6.966	1002.840
bp7j	141	643	41193	83	83	1674	618	7	836	7.610	1770.290
bp7k	140	642	40268	83	83	912	456	2	455	6.669	933.364
bp7l	141	643	40892	83	83	1126	563	6	562	7.384	1176.710
bp7m	141	643	41322	83	83	1262	631	6	630	7.961	1335.260
bp7n	141	643	40838	83	83	1196	598	2	597	6.786	1120.000
bp7o	145	647	43397	83	83	1084	542	3	541	7.408	1108.440
bp7p	142	644	41667	83	83	1212	606	4	605	6.868	1185.170
bp7q	144	646	42116	83	83	2236	635	10	1117	7.632	2299.720
bp7r	145	647	43235	83	83	2746	798	20	1372	8.593	2958.300
bp7s	138	640	39957	83	83	664	332	2	331	6.620	651.677
bp7t	136	638	38154	83	83	636	313	3	317	6.088	580.249
Column avg.				83.00	83.00	1240.40	520.35	5.35	619.20	7.11	1237.72

Table 13: binpack7 instances - Benders Decomposition

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Price (s)	Solve (s)
bp7a	134	636	37895	83	83	293	146	197.060	557.717
bp7b	140	642	40648	83	83	345	172	247.573	952.199
bp7c	139	641	40490	83	83	361	180	263.842	961.816
bp7d	142	644	41058	83	83	395	197	341.171	1188.720
bp7e	134	636	37757	83	83	213	106	116.683	352.838
bp7f	145	647	43028	83	83	611	295	570.238	2142.400
bp7g	138	640	39188	83	83	335	167	196.613	752.845
bp7h	137	639	39085	83	83	309	154	190.410	735.178
bp7i	139	641	40328	83	83	435	217	313.225	1203.160
bp7j	141	643	41193	83	83	401	200	359.847	1318.840
bp7k	140	642	40268	83	83	337	168	247.277	908.949
bp7l	141	643	40892	83	83	443	221	312.366	1296.570
bp7m	141	643	41322	83	83	441	220	324.805	1295.150
bp7n	141	643	40838	83	83	385	192	266.918	1018.470
bp7o	145	647	43397	83	83	537	268	495.756	2045.120
bp7p	142	644	41667	83	83	403	201	290.112	1199.990
bp7q	144	646	42116	83	83	387	193	333.500	1273.290
bp7r	145	647	43235	83	83	535	267	517.882	2004.780
bp7s	138	640	39957	83	83	531	265	385.627	1555.900
bp7t	136	638	38154	83	83	439	219	297.010	1130.800
Column avg.				83.00	83.00	406.80	202.40	313.40	1194.74

Table 14: binpack7 instances - Branch-and-Price [2]

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{ip}^*	Nodes	Level	Benders Cuts	Other Cuts	Price (s)	Solve (s)
bp8a	190	692	62337	167	167	2742	1371	5	1370	22.650	4774.120
bp8b	192	694	63790	167	167	3226	1176	4	1612	25.342	5385.220
bp8c	190	692	63433	167	167	3532	1766	7	1765	24.706	5250.980
bp8d	199	701	67960	167	167	2342	1171	1	1170	24.496	3810.650
bp8e	195	697	65515	167	167	2494	1247	4	1246	25.873	3969.240
bp8f	195	697	65314	167	167	3160	1580	7	1579	25.420	5792.400
bp8g	196	698	65987	167	167	1208	604	2	603	23.350	2080.780
bp8h	192	694	63804	167	167	2966	1483	4	1482	25.548	4607.650
bp8i	196	698	65567	167	167	3394	1697	5	1696	22.685	5481.910
bp8j	189	691	62452	167	167	3014	1507	5	1506	25.071	4632.540
bp8k	190	692	62770	167	167	572	286	0	285	22.348	861.067
bp8l	195	697	65263	167	167	2410	1205	4	1204	25.272	3765.400
bp8m	189	691	62097	167	167	3254	1627	8	1626	26.465	5347.340
bp8n	198	700	66788	167	167	2408	1204	3	1203	26.219	4255.190
bp8o	203	705	69699	167	167	3926	1963	5	1962	25.985	6720.880
bp8p	197	699	66678	167	167	2152	1076	4	1075	24.541	3508.960
bp8q	198	700	66895	167	167	3156	1578	6	1577	23.814	5507.770
bp8r	196	698	65555	167	167	3052	1526	7	1525	24.163	4922.850
bp8s	193	695	64091	167	167	2194	1097	0	1096	23.701	3774.760
bp8t	192	694	63484	167	167	2928	1464	4	1463	26.414	4397.070
Column avg.				167.00	167.00	2706.50	1331.40	4.25	1352.25	24.70	4442.34

Table 15: binpack8 instances - Benders Decomposition

Instance	Products	Nodes	Edges	Z_{lp}^*	Z_{tp}^*	Nodes	Level	Price (s)	Solve (s)
bp8a	190	692	62337	167	167	703	351	930.469	4224.010
bp8b	192	694	63790	167	167	735	367	1029.260	4785.730
bp8c	190	692	63433	167	167	943	471	1421.110	6449.990
bp8d	199	701	67960	172	172	975	487	1466.670	7197.900
bp8e	195	697	65515	170	170	1021	510	1719.040	7176.630
bp8f	195	697	65314	167	167	841	420	1256.310	5794.760
bp8g	196	698	65987	167	167	745	372	1012.270	5087.650
bp8h	192	694	63804	168	168	1177	567	1429.230	7177.330
bp8i	196	698	65567	167	167	841	420	1338.960	5979.840
bp8j	189	691	62452	167	167	845	422	1198.790	5665.170
bp8k	190	692	62770	167	167	1069	534	1706.170	7011.920
bp8l	195	697	65263	171	171	991	495	1592.870	7173.250
bp8m	189	691	62097	167	167	747	373	921.158	4152.980
bp8n	198	700	66788	167	167	909	454	1338.510	6287.870
bp8o	203	705	69699	167	167	679	339	1047.110	4640.550
bp8p	197	699	66678	173	173	1113	556	1053.690	7222.670
bp8q	198	700	66895	167	167	1025	512	2036.910	7141.220
bp8r	196	698	65555	167	167	1001	500	1705.930	7087.380
bp8s	193	695	64091	167	167	795	397	1086.490	5038.840
bp8t	192	694	63484	167	167	701	350	871.090	4267.940
Column avg.				167.95	167.95	892.80	444.85	1308.10	5978.18

Table 16: binpack8 instances - Branch-and-Price [2]